MATH 31 - using Synthetic DIVISION to find FACTORS QUICKLY.

When we examine the REMAINDER THEOREM CAREFULLY, we can see that if the REMAINDER is zero, then $(x-a)$ is a factor!

FOR EXAMPLE: $P(x) = x^3-2x^2-9x+18 = \underbrace{(x-2)(x^2-9)}_{\text{verify this for yourself}}$

verify by synthetic Division:

$$\begin{array}{r|rrrr} & 1 & -2 & -9 & +18 \\ \hline 2) & 1 & 0 & -9 & 0 \end{array} \leftarrow$$

Remainder is zero! $x-2$ is a FACTOR!

Remainder Theorem say:

$(x-a)\,Q(x) + R = P(x)$

if $R=0$, then $\underline{P(x) = (x-a)Q(x) \quad \text{- FACTOR Theorem}}$

So this means that we can quickly find factors (if there are any) by using synthetic division.

Consider $P(x) = 2x^3-5x^2-14x+8$ and try to find a factor by trying some values.

$$\begin{array}{r|rrrr} & 2 & -5 & -14 & 8 \\ \hline -2 & 2 & -9 & 4 & 0 \\ -1 & 2 & -7 & -7 & 15 \\ 1 & & & & \end{array}$$

2.5.4, we will see how to use the RATIONAL ZEROS THEOREM TO HELP SELECT CANDIDATES.

FOR NOW I WILL TRY $\pm1, \pm2$, etc. & hope I hit on a zero, AND I DO!

So now I know that I have factored my original polynomial as follows:

$$2x^3-5x^2-14x+8 = (x+2)(2x^2-9x+4) \quad +0$$

once I reduce the degree to 2nd, I can try to factor this quadratic by usual F.O.I.L. method.

$$2x^2-9x+4 = (2x-1)(x-4)$$

So we have been able to completely factor $P(x) = (x+2)(2x-1)(x-4)$ and its zeros are $-2, \frac{1}{2}, 4$.