

① Detailed Example - Solving a rational equation (an equation with unknowns in denominator)

SOLVE: $\frac{2}{x^2-1} - \frac{x-1}{x+1} = \frac{3}{5x-5}$

with non-integer answers and non-functional answers. i.e. IRRATIONAL ANSWERS

STEP 1: FACTOR ALL DENOMINATORS

$$\frac{2}{(x+1)(x-1)} - \frac{x-1}{x+1} = \frac{3}{5(x-1)}$$

STEP 2: FIND LCD, & MULTIPLY BOTH SIDES OF EQUATION BY LCD

LCD = $5(x+1)(x-1)$

$$5(x+1)(x-1) \left[\frac{2}{(x+1)(x-1)} - \frac{x-1}{x+1} \right] = \frac{3 \cdot 5(x+1)(x-1)}{5(x-1)}$$

NOTE: ABOVE STEP CAN BE DONE MENTALLY WITH A BIT OF PRACTICE THE NET EFFECT IS THAT ALL DENOMINATORS WILL BE ELIMINATED

STEP 3: AFTER MULTIPLICATION WE HAVE:

$$\begin{aligned} 2 \cdot 5 & - 5(x-1)(x-1) = 3(x+1) \\ \text{OR } 10 & - 5(x^2 - 2x + 1) = 3x + 3 \end{aligned}$$

NOW SIMPLIFY THE RESULTING EQUATION

OR $10 - 5x^2 + 10x - 5 = 3x + 3$ - WATCH THOSE SIGNS!

STEP 4: $10 - 5x^2 + 10x - 5 = 3x + 3$

$$-5x^2 + 10x + 5 = 3x + 3$$

$$-5x^2 + 7x + 2 = 0$$

(change all signs so coef. a > 0)
(Less confusing)

$$5x^2 - 7x - 2 = 0$$

IF YOU END UP WITH A QUADRATIC EQUATION
STEP 5: SOLVE SIMPLIFIED EQUATION

THIS EQUATION DOES NOT FACTOR! VERIFY THIS FOR YOURSELF BY COMPUTING THE DISCRIMINANT $b^2 - 4ac$

$x = \frac{7 \pm \sqrt{89}}{10}$. $\sqrt{89}$ cannot be simplified! (Why NOT?)

You can give approximate decimal answers though. $x \approx \frac{7 \pm 9.4}{10}$ or $x \approx 1.6$ or 0.24

Do an approximate check using the decimal approximations. (OR check at least

IS $\frac{2}{(1.6)^2-1} - \frac{1.6-1}{1.6+1} \stackrel{?}{\approx} \frac{3}{5(1.6)-5}$ USE ORIGINAL EQUATION (IF AN ANSWER yields zero in denominator, you MUST discard it.)

You will get $1.28 - 0.23 = 1.05$ and 1 on the right.

$1.05 \approx 1$, so answer looks correct. IT.