MATH 31 – POLYNOMIAL DIVISION

POLYNOMIAL LONG DIVISION

$$
\begin{array}{r}
x^2 - x - 3 \\
x-5 \,\big)\, \overline{x^3 - 6x^2 + 2x + 1} \\
\underline{x^3 - 5x^2} \qquad\quad \\
-x^2 + 2x \quad\;\; \\
\underline{-x^2 + 5x} \quad\;\; \\
-3x + 1 \\
\underline{-3x + 15} \\
\text{(Rem.)}\; -14
\end{array}
$$

DIVIDEND is $P(x)$
$= x^3 - 6x^2 + 2x + 1$

DIVISOR is $x - 5$

QUOTIENT is $x^2 - x - 3$

Remainder is -14

SAME PROBLEM DONE BY "SYNTHETIC" DIVISION WHICH OMITS VARIABLES.

WRITE THE COEFFICIENTS OF $P(x)$ AND WRITE THE a of $(x-a)$, the divisor, at left.

	1	-6	2	1
5	↓	$5\cdot1-6$	$(5)(-1)+2$	$5(-3)+1$
	1	-1	-3	-14

← learn to do this mentally

OBSERVE! THE NEW ROW GIVES the QUOTIENT'S COEFFICIENTS and the remainder.

$$1\cdot x^2 - 1\cdot x - 3 = Q(x)$$

and $R = -14$

REMAINDER THEOREM SAYS

DIVIDEND = (DIVISOR)(QUOTIENT) + REMAINDER

so

$$x^3 - 6x^2 + 2x + 1 = (x-5)(x^2 - x - 3) - 14$$

SYNTHETIC DIVISION is the method of choice when we want to find & plot several points for a specific polynomial. The last number, the remainder is $P(a)$. So for example, we want to compute this table of points

a	$P(a)$
-3	-20
-2	-11
-1	-2
0	1
1	-2
2	-11
3	-20
10	421

NOTICE! EVEN THOUGH THESE y-values < 0, by the time $x = 10$, $P(x) = 421$ which is a large POSITIVE number.

MAKE A SYNTHETIC DIVISION CHART

a	1	-6	2	1
3	1	-3	-7	-20
2	1	-4	-6	-11
1	1	-5	-3	-2
0	obvious!			1
1	1	-5	-3	-2
2	1	-4	-6	-11
3	1	-3	-7	-20
10	1	+4	42	421

NOTICE $P(x) \to \infty$