EXAMPLE A OF ANALYSIS & GRAPHING A POLYNOMIAL

$$P(x) = -x^5 - x^4 + 10x^3 + 10x^2 - 9x - 9$$

END BEHAVIOR: as $x \to \infty$, $P(x) \to -\infty$
as $x \to -\infty$, $P(x) \to \infty$

✓ Descartes Rule of Signs tells us that since there are 2 sign changes in $P(x)$, there are either 2 or 0 real zeros* (Positive)

* These come in pairs because a pair of zeros represents where a "blip" occurs

$P(-x) = +x^5 - x^4 - 10x^3 + 10x^2 + 9x - 9$

$P(-x)$ has 3 sign changes so there are 3 or 1 real zeros† < 0. (negative)

✓ RATIONAL ZEROS THEOREM tells us that possible rational zeros are

$\frac{p}{q}$ is $\{\pm 1, \pm 3, \pm 9\}$ — lead coef. is -1, so q is only ± 1.

SYNTHETIC DIVISION CHART

	−1	−1	10	10	−9	−9	
9	−1	−10	−80	−710	large neg.	large negative	**
3	−1	−4	−2	4	3	0	← ZERO – 1st rational zero
1	−1	−2	8	18	9	0	← 2nd rational zero
0						−9	Y-intercept
−1	−1	0	10	0	−9	0	← 3rd rational zero
−3	−1	2	4	−2	−3	0	← 4th rational zero
−9	−1	8	−62	large +	large −	large +	alternating signs means a Lower Bound.

** SINCE the leading coefficient is <0, the boundedness theorem is reversed, i.e. A ROW of all negative numbers, means that 9 is an UPPER BOUND on all zeros

THIS is because $-P(x)$ would have all positive numbers in Row where $x = 9$.

† If a real irrational zero exists, it will have the form $a \pm b\sqrt[n]{m}$ (conjugate pairs)

We have found only 4 of the rational zeros. There must be another negative rational zero. So we shall use the depressed polynomial and look again at -3 or -1 to see if there is a double zero. →

	−1	0	10	0	−9	
−1	−1	+1	9	−9	0	← Double zero

Use the $x=-1$ depressed polynomial →

(OVER)